



BWPA Maths Calculation Policy

Maths Mastery

At the depth of the mastery approach to the teaching of mathematics is the belief that **all children have the potential to succeed**. They should have access to the same curriculum content and, rather than being extended with new learning, they should **deepen their conceptual understanding by tackling challenging and varied problems**. Similarly, with calculation strategies, children must not simply rote learn procedures but demonstrate their understanding through the use of concrete materials and pictorial representations. This policy outlines the different calculation strategies that should be taught and used in EYFS through to Year 6 in line with the requirements of the 2014 Primary National Curriculum.

Mathematical Language

The 2014 Primary National Curriculum is explicit in articulating the importance of children using the correct mathematical language as part of their learning (reasoning). Indeed, in certain year groups, the non-statutory guidance highlights the requirements for children to extend their language around certain concepts. It is therefore essential that teaching the strategies outlined in this policy is accompanied by the use of appropriate and precise mathematical vocabulary. New vocabulary should be introduced in a suitable context (for example, with relevant real objects, apparatus, pictures or diagrams) and explained carefully. High expectations of the mathematical language used are essential, with teachers only accepting what is correct. The agreed list of terminology is above each mathematical operation in this policy.

The quality and variety of language that pupils hear and speak are key factors in developing their mathematical vocabulary and presenting a mathematical justification, argument or proof.
2014 Maths Programme of Study

How to use the policy


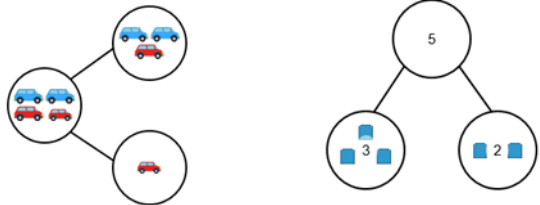


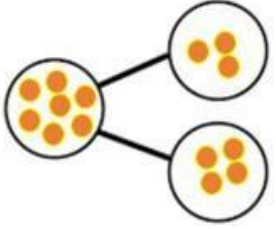

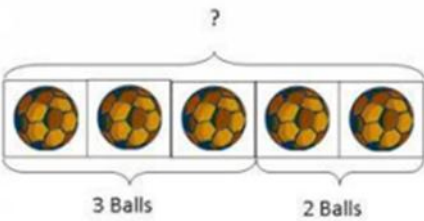
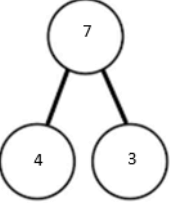

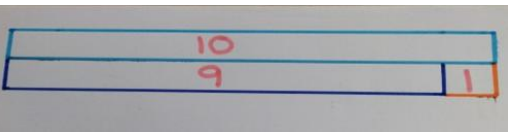
This policy is a guide for all teaching staff. It is purposefully set out as a progression of mathematical skills and not into year group phases to encourage a flexible approach to teaching and learning. It is expected that teachers will use their professional judgement as to when consolidation of existing skills is required or if to move onto the next concept. However, the focus always **must remain on breadth and depth rather than accelerating through concepts**. Children should not be extended with new learning before they are ready, they should deepen their conceptual understanding by tackling challenging and varied problems.

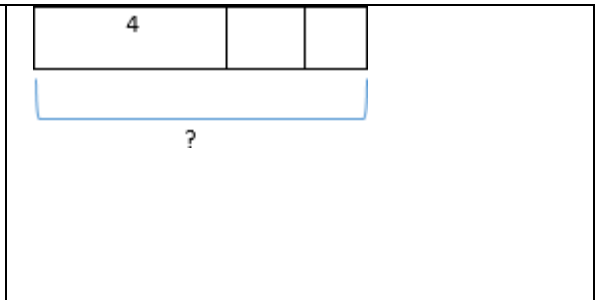
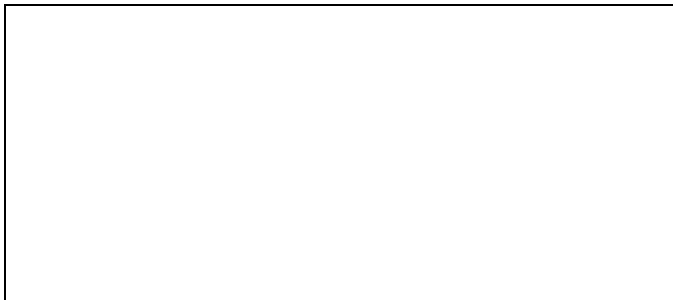
For each of the four rules of number, different strategies are laid out, together with examples of what concrete materials can be used and how, along with suggested pictorial representations. The main concrete materials to be used within all year groups are Dienes, PV counters and Cuisenaire rods. The principle of the concrete-pictorial-abstract (CPA) approach (make it, draw, write it) is for children to have a true understanding by mastering all these three phrases within each mathematical concept.

	EYFS	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	<p>Combining two parts to make a whole: part whole model</p> <p>Start with the bigger number and count on</p> <p>Regrouping to make 5 using the five frame</p>	<p>Combining two parts to make a whole: part whole model</p> <p>Start with the larger number and count on</p> <p>Regrouping to make 10 using the ten frame</p>	<p>Adding 3 single digit numbers</p> <p>Use of base 10 to combine two numbers</p>	<p>Column method - regrouping</p> <p>Using PV counters (up to 3 digits)</p>	<p>Column method - regrouping (up to 4 digits)</p>	<p>Column method - regrouping</p> <p>PV with decimals</p>	<p>Column method with regrouping</p> <p>Abstract methods</p>
Subtraction	<p>Counting back</p> <p>Taking away ones</p> <p>Part whole model</p> <p>Making 5 using the five frame</p>	<p>Counting back</p> <p>Taking away ones</p> <p>Find the difference</p> <p>Part whole model</p> <p>Make 10 using the 10 frame</p>	<p>Counting back</p> <p>Find the difference</p> <p>Part whole model</p> <p>Make 10</p> <p>Use of base 10</p>	<p>Column method with regrouping</p> <p>(up to 3 digits using PV counters)</p>	<p>Column method with regrouping</p>	<p>Column method with regrouping</p> <p>Abstract for whole numbers</p> <p>PV with decimals</p>	<p>Column method with regrouping</p> <p>Abstract methods</p>

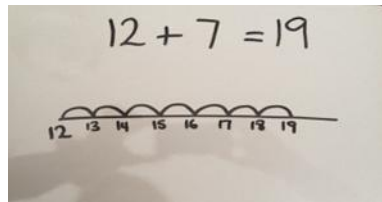
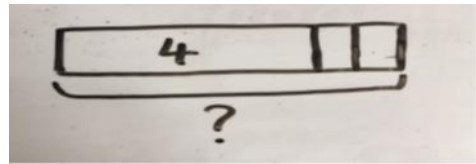
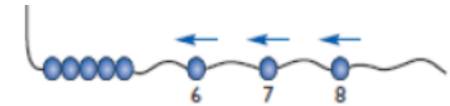
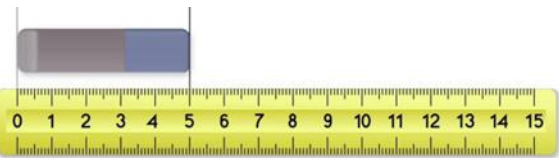
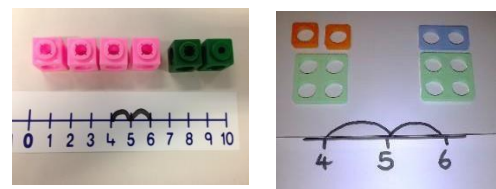
Addition-

Key language which should be used: sum, total, parts and wholes, plus, add, altogether, more than, 'is equal to' 'is the same as'

Concrete	Pictorial	Abstract
<p>Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears etc)</p>  <p>Part whole model</p>   <p>Link to addition - use the part whole model to explain the inverse between addition and subtraction</p> 	<p>Pictorial</p>   <p>A bar model which encourages the children to count on</p> 	<p>Abstract</p> <p>$4 + 3 = 7$ (four is a part, 3 is a part and the whole is seven)</p>   <p>Use the part-part whole diagram as shown above to move into the abstract.</p> <p>$4 + 3 = 7$ $10 = 6 + 4$</p> 

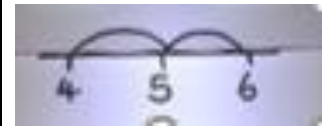


Counting on using number lines by using cubes or numicon



Start at the larger number on the number line and count on in ones or in one jump to find the answer.

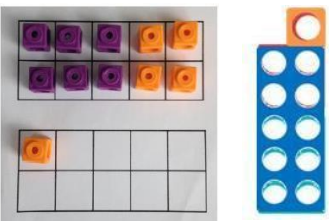
The abstract number line:
What is 2 more than 4? What is the sum of 4 and 4? What's the total of 4 and 2?
 $4 + 2$



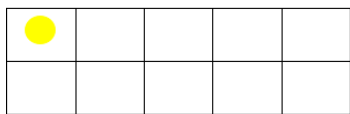
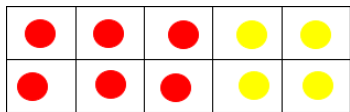
Regrouping to make 10 by using ten frames and counters/cubes or using numicon:

$6 + 5$ becomes $6 + 4 = 10$ $10 + 1 = 11$

This then moves on to missing number questions worked out in the same way $5 + _ = 12$



Children to draw the ten frame and counters/cubes



Children will add by bridging through 10 mentally.

Children to develop an understanding of equality e.g $6 + \square = 11$ and

$6 + 5 = 5 + \square$ $6 + 5 = \square + 4$

TO + O using base 10. Continue to develop understanding of partitioning and place value $41 + 8$

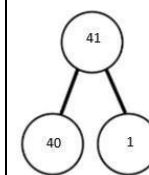
This would move onto exchanging tens for a rod of 10. Show how to represent on calculation as each step is taken with the concrete. Develop this into missing number questions.



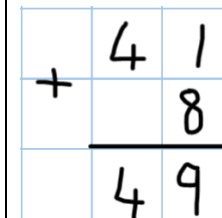
Children to represent the concrete using a particular symbol e.g. lines for tens and crosses for ones. When exchanging occurs children can group the ten ones or cross them out and exchange for a ten.



$41 + 8$



$1 + 8 = 9$
 $40 + 9 = 49$

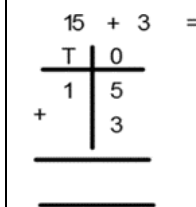
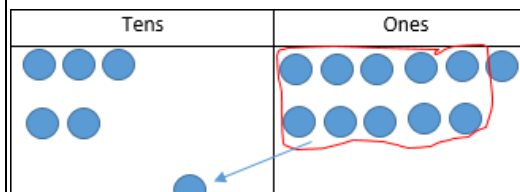


TO + TO using base 10. Continue to develop understanding of partitioning and place value and use this to support addition. Begin with no exchanging. Then move into exchanging $36 + 25$

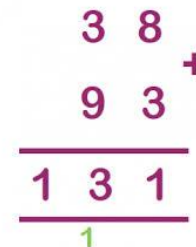
	Tens	Ones
+		
=		

Here 10 ones have been exchanged for one ten.

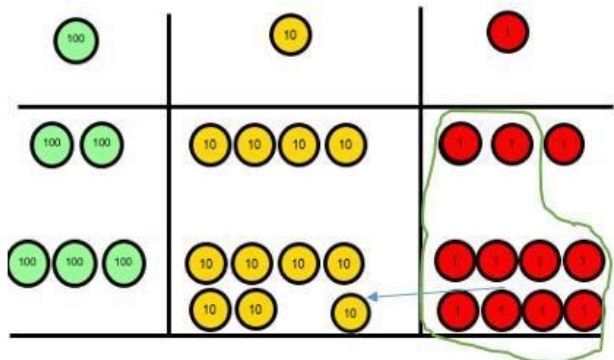
This could be done one of two ways:



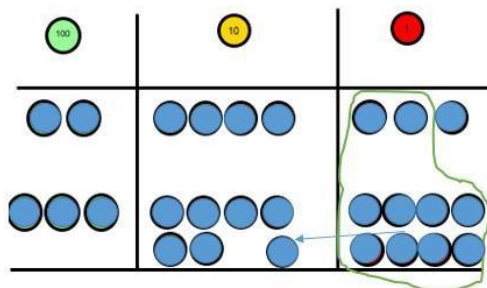
Formal method:



Use of place value counters to add HTO + TO, HTO + HTO etc. once the children have had practice with this, they should be able to apply it to larger numbers and the abstract

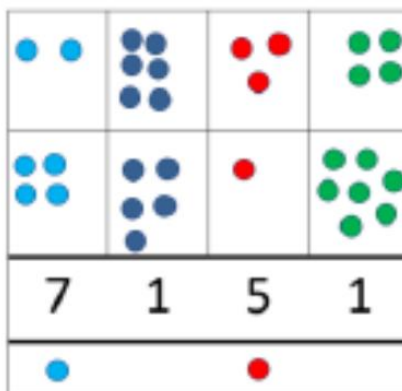


Children to represent the counters e.g. like the image below

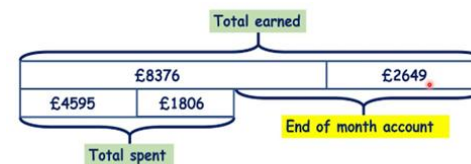


If the children are completing a word problem, draw a bar model to represent what it's asking them to do

?	
243	368



$$\begin{array}{r} 427 \\ + 363 \\ \hline 790 \\ 1 \end{array}$$



As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here.

$$\begin{array}{r} \text{£ } 23.59 \\ + \text{£ } 7.55 \\ \hline \text{£ } 31.14 \\ 1 \quad 1 \quad 1 \end{array}$$

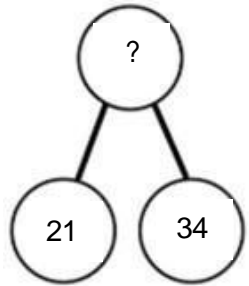
$$\begin{array}{r} 72.8 \\ + 54.6 \\ \hline 127.4 \\ 11 \end{array}$$

$$3.46 + 3.792 =$$

$$\begin{array}{r} 3.460 \\ 3.792 \\ \hline 7.252 \end{array}$$

Zero used as a place value holder.

Fluency variation, different ways to ask children to solve 21+34:



Sam saved £21 one week and £34 another. How much did he save in total?

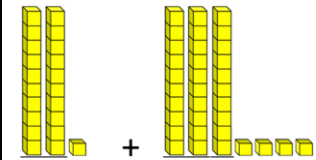
21+34=55. Prove it! (reasoning but the children need to be fluent in representing this)

$$\begin{array}{r} 21 \\ +34 \\ \hline \end{array}$$

$$21 + 34 =$$

$$\square = 21 + 34$$

What's the sum of twenty one and thirty four?

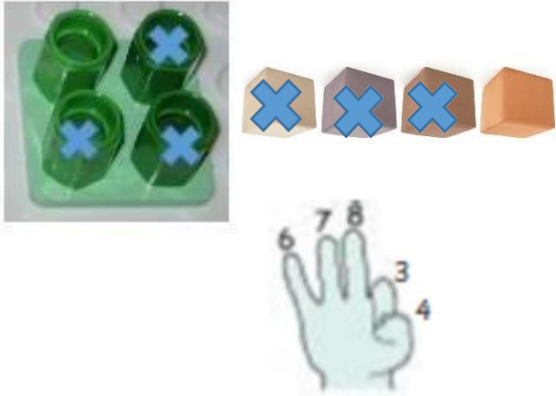

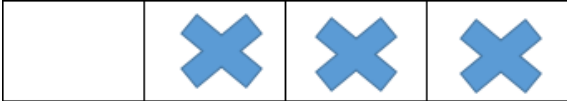
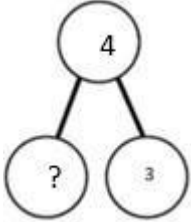
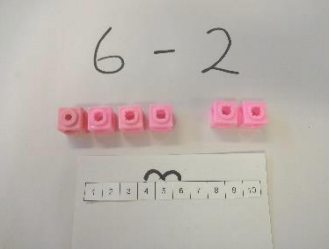
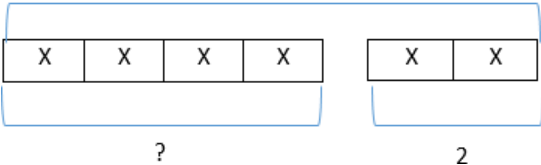



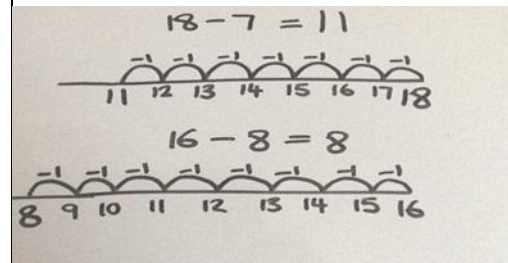
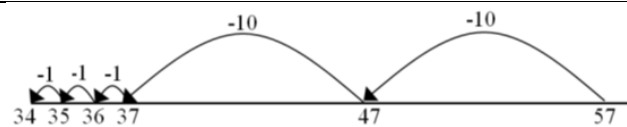
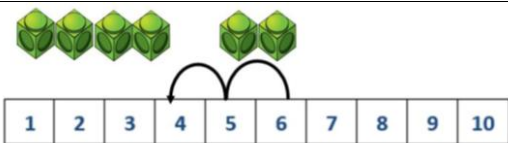
Always use missing digit problems too:

Tens	Ones

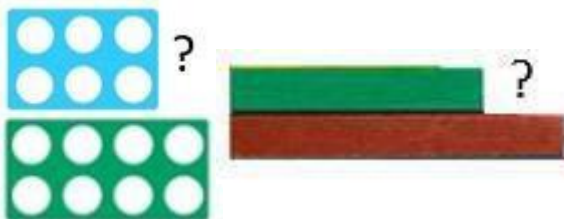
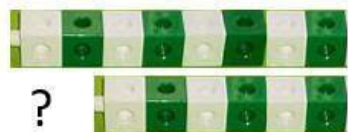
Subtraction-

Key language which should be used: take away, less than, the difference, subtract, minus, fewer, decrease, '7 take away 3, the difference is four'

Concrete	Pictorial	Abstract				
<p>Physically taking away and removing objects from a whole rather than crossing out- children will physically remove the objects. Move this onto missing numbers</p> <p>$4 - 3 = 1$</p> 	<p>Children to draw the concrete resources they are using and cross out.</p>  <p>Use of the bar model:</p> 	<p>$4 - 3 =$</p> <p><input type="text"/> = $4 - 3$</p> <table border="1" data-bbox="1415 594 1726 672"> <tr> <td colspan="2">4</td> </tr> <tr> <td>3</td> <td>?</td> </tr> </table> 	4		3	?
4						
3	?					
<p>Counting back (using number lines or number tracks)</p> 	<p>Children to represent what they see pictorially e.g.</p> <p>6</p> 	 <p>Number in your head and count back using a number line as a visual aid.</p>				



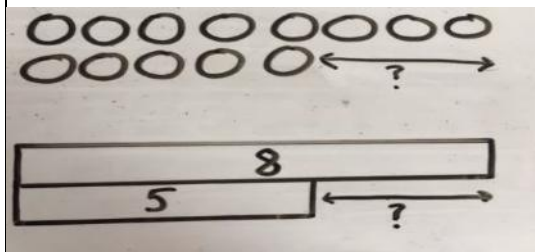
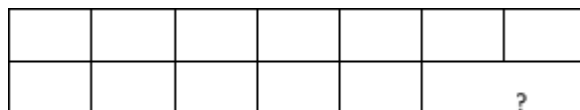
Finding the difference (using cubes, numicon or Cuisenaire rods, other objects can also be used)



Children to draw the cubes/other concrete objects which they have used

XXXXXXXXX
XXXXXXX

Use of the bar model



Find the difference between 8 and 6.
 $8 - 6$, the difference is ?

Children to explore why

$$9 - 6 =$$

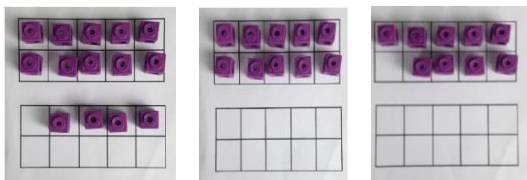
$$8 - 5 =$$

$$7 - 4 =$$

have the same difference.

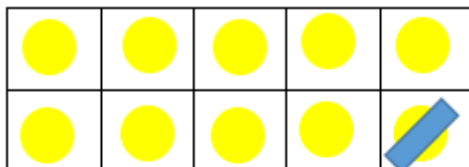
Children to also explore why
 $9 - 7 = 8 - 6$ (the difference, of each digit, has changed by 1 so the difference is the same)

Making 10 (using numicon or ten frames)
 $14 - 5$ becomes $14 - 4 = 10$ then take one more away to gain answer of 9.



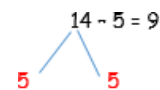
Carry this out with missing number questions
 e.g. $16 - _ = 7$

Children to present the ten frame pictorially

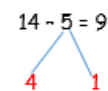


$14 - 5 = 9$ You also want children to see related facts e.g. $15 - 9 = 5$

Children to represent how they have solved it e.g.



$14 - 5 = 9$ 14 is made up of 5, 5 and 4 so I can subtract one 5 to be left with 4 and 5

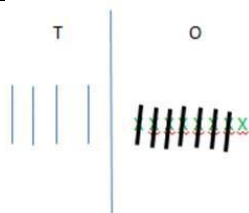


$14 - 5 = 9$ 5 is made up of 4 and 1 so I can subtract 4 to make 10 and then 1 to get to 9

Column method (using base 10)
 $48 - 7$

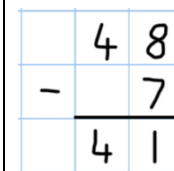


Develop this into missing number questions.



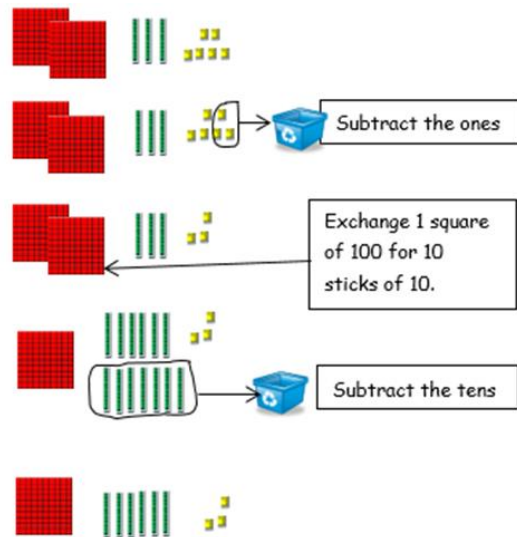
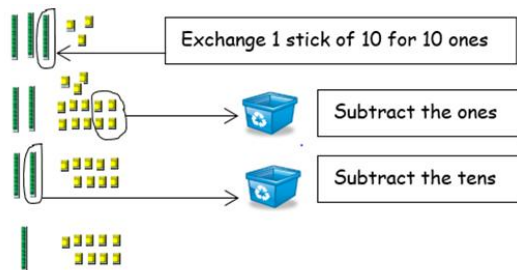
Develop this into missing number questions.

$48 - 7 =$



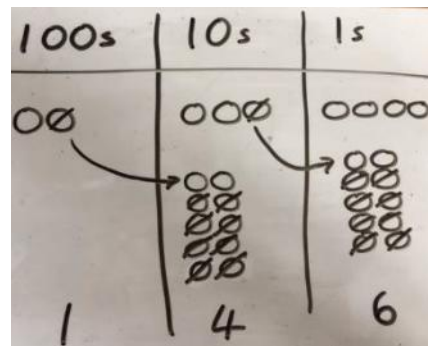
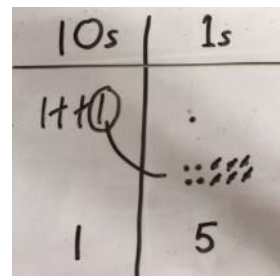
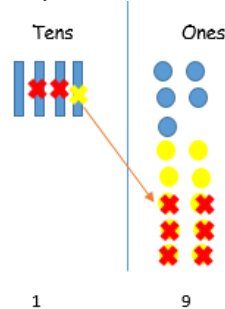
Develop this into missing number questions.

Column method (using base 10 and having to exchange)

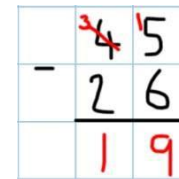


Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

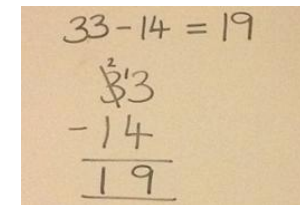
Represent the base 10 pictorially



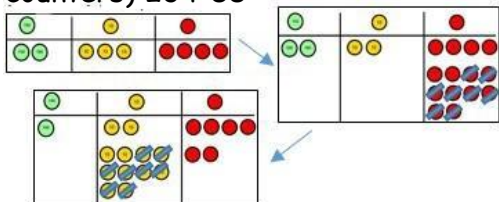
It's crucial that the children understand that when they have exchanged the 10 they still have 45. $45 = 30 + 15$



391	
186	?



Column method (using place value counters) 234-88



Once the children have had practice with the concrete, they should be able to apply it to any subtraction.

Like the other pictorial representations, children to represent the counters.

Move onto larger numbers with many exchanges. Then decimal numbers including money.

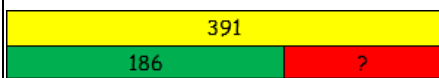
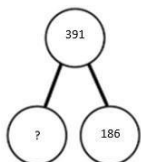
$$\begin{array}{r} 341 - 183 \\ \cancel{3}4\overset{13}{1} \\ - 183 \\ \hline 158 \end{array}$$

$$\begin{array}{r} 4.63 - 2.91 = \\ \cancel{4}.63 \\ - 2.91 \\ \hline 1.72 \end{array}$$

$$\begin{array}{r} 7.35 - 2.1 - 1.675 = \\ 1.675 + 2.100 = 3.775 \\ \cancel{3}.775\overset{0}{0} \\ - 3.775 \\ \hline 0 \end{array}$$

Zero used as place holder

Fluency variation, different ways to ask children to solve 391-186:



Raj spent £391, Timmy spent £186. How much more did Raj spend?

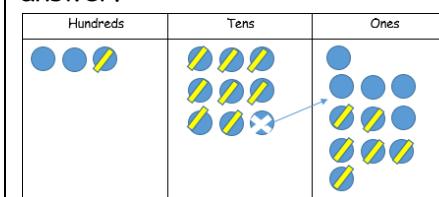
I had 391 metres to run. After 186 I stopped. How many metres do I have left to run?

$$\begin{array}{r} 391 - 186 \\ = 391 - 186 \end{array}$$

$$\begin{array}{r} 391 \\ - 186 \\ \hline \end{array}$$

Find the difference between 391 and 186
Subtract 186 from 391.
What is 186 less than 391?

What's the calculation? What's the answer?

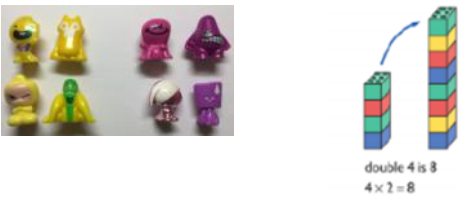
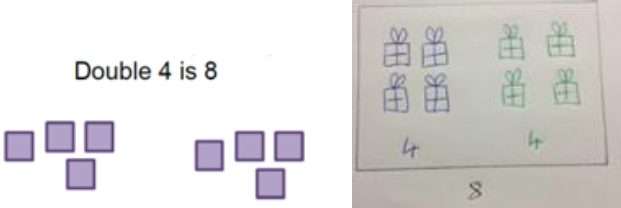
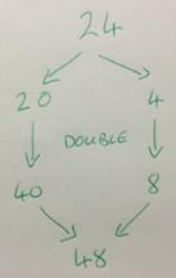

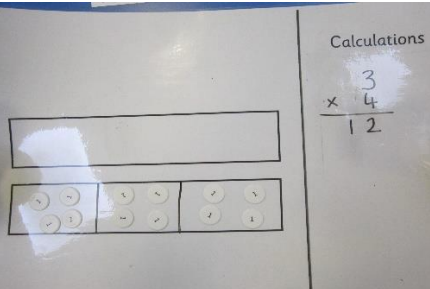



$$\begin{array}{r} 39\ \square \\ - \square\square 6 \\ \hline \square 0 5 \end{array}$$

	EYFS	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Multiplication	ELG: solve problems, including doubling	Doubling	Arrays – showing commutative multiplication	Arrays 2d x 1d	Column multiplication – introduced with place value counters (2 and 3 digit multiplied by 1 digit)	Column multiplication Abstract only but might need a repeat of year 4 first (up to 4 digit numbers multiplied by 1 or 2 digits)	Column multiplication Abstract methods
Division	ELG: solve problems, including halving and sharing	Sharing objects into groups	Division as grouping Division within arrays – linking to multiplication Repeated subtraction	Division with a remainder 2d divided by 1 d using base 10 or place value counters	Division with a remainder Short division (up to 3 digits by 1 digit – concrete and pictorial)	Short division (up to 4 digits by a 1 digit number including remainders)	Short division Long division with place value counters (up to 4 digits by a 2 digit) Children should exchange into the tenths and hundredths column too.

Multiplication-

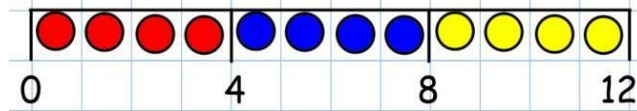
Key language which should be used: double times, multiplied by, the product of, groups of, lots of, 'is equal to' 'is the same as'

Concrete	Pictorial	Abstract
<p>Doubling as a strategy</p>  <p>double 4 is 8 $4 \times 2 = 8$</p>	<p>Double 4 is 8</p> 	<p>Partition a number and then double each part before recombining it back together.</p> 
<p>Repeated grouping/repeated addition 3×4 or 3 lots of 4</p> <p>Counting in multiples (groups) supported concrete objects in equal group.</p>  	<p>Use of a bar model to draw dots</p> 	<p>3×4</p> <p>$4 + 4 + 4$</p>

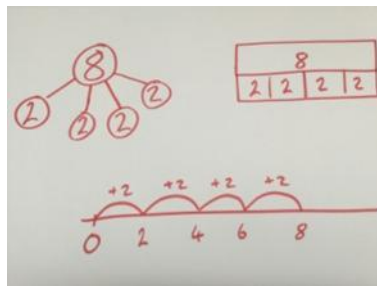
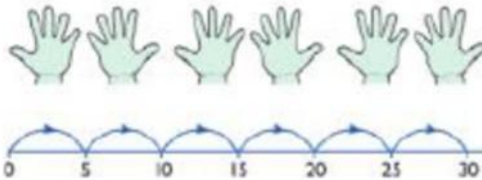
Use number lines to show repeated groups- 3×4



Represent this pictorially alongside a number line e.g:

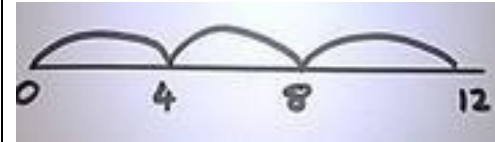


Use a number line or pictures to continue support in counting in multiples.



Abstract number line

$$3 \times 4 = 12$$



Write sequences with multiples of numbers:

2, 4, 6, 8, 10, 12,

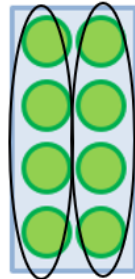
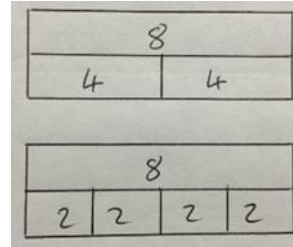
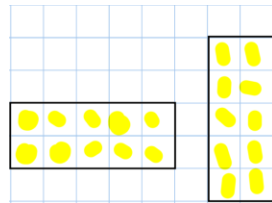
5, 10, 15, 20, 25, 30

Use arrays to illustrate commutativity
(counters and other objects can also be used)

$$2 \times 5 = 5 \times 2$$

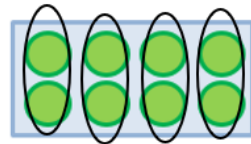


Children to draw the arrays



4 2 times

$$4 \times 2 = 8$$



2 4 times

$$2 \times 4 = 8$$

Children to be able to use an array to write a range of calculations e.g.

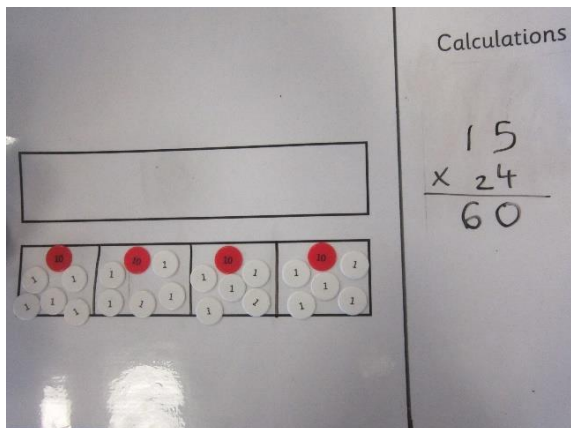
$$2 \times 5 = 10$$

$$5 \times 2 = 10$$

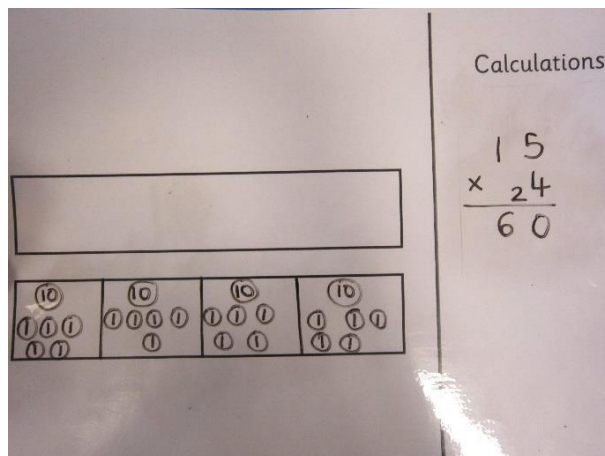
$$2 + 2 + 2 + 2 + 2 = 10$$

$$5 + 5 = 10$$

Partition to multiply 4×15 using place value counters on a bar model.



Draw place value counters on the bar model.



Children to be encouraged to show the steps they have taken

$$4 \times 15$$

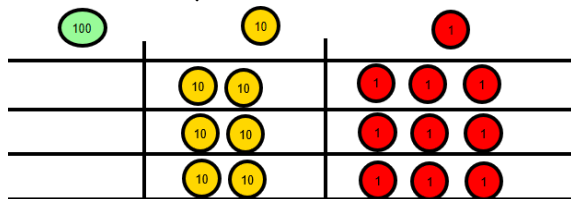
$$\begin{array}{r} 15 \\ \times 24 \\ \hline 60 \end{array}$$

$4 \times 5 = 20$
 $4 \times 10 = 40$
 $40 + 20 = 60$

This is a step before formal written method.

Formal column method with place value counters or base 10 (at the first stage-no exchanging) 3×23

Make 23, 3 times. See how many ones, then how many tens



Children to represent the counters in a pictorial way

Tens	Ones
6	9

Children to record what it is they are doing to show understanding

$$3 \times 23$$

$$\begin{array}{r} 23 \\ \times 3 \\ \hline 69 \end{array}$$

$3 \times 3 = 9$
 $3 \times 20 = 60$
 $60 + 9 = 69$

$$\begin{array}{r} 23 \\ \times 3 \\ \hline 69 \end{array}$$

Formal column method with place value counters (children need this stage, initially, to understand how the column method works)

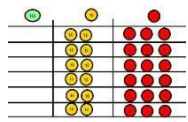
Children to represent the counters/base 10, pictorially e.g. the image below.

$$6 \times 23$$

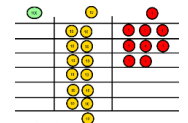
$$\begin{array}{r} 23 \\ \times 6 \\ \hline 138 \end{array}$$

$6 \times 3 = 18$
 $6 \times 20 = 120$
 $120 + 18 = 138$

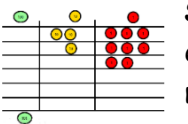
6×23



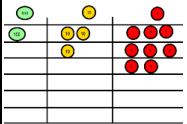
Step 1: get 6 lots of 23



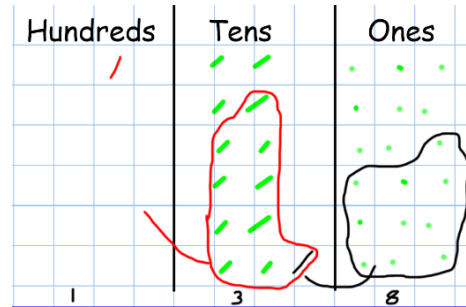
Step 2: 6×3 is 18. Can I make an exchange? Yes! Ten ones for one ten....



Step 3: 6×2 tens and my extra ten is 13 tens. Can I make an exchange? Yes! Ten tens for one hundred...



Step 4- what do I have I each column?



The aim is to get to the formal method but the children need to understand how it works.

$$\begin{array}{r} 24 \\ \times 3 \\ \hline 72 \\ \hline \end{array}$$

Here each step that is taken with the concrete needs showing on the written calculation alongside. E.g. as an exchange is made show how that would look on the calculation.

When children start to multiply $3d \times 3d$ and $4d \times 2d$ etc, they should be confident with the abstract:

To get 744 children have solved 6×124

To get 8680 they have solved 70×124

When exchanging in the first calculation, the exchanged number goes above the line.

When children start to multiply the tens or hundreds, they must cross out the exchanging from the previous calculation and write in the new exchanging.

$$\begin{array}{r}
 124 \\
 \times 70 \\
 \hline
 8680 \\
 \times 700 \\
 \hline
 86800 \\
 \hline
 94240
 \end{array}$$

Answer: 9424

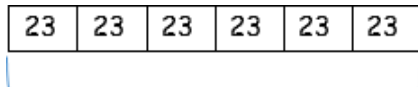
Move onto decimals.

Handwritten examples of decimal multiplication:

$$\begin{array}{r}
 1.27 \\
 \times 3 \\
 \hline
 3.81
 \end{array}$$

$$\begin{array}{r}
 1.27 \\
 \times 15 \\
 \hline
 6.35 \\
 12.70 \\
 \hline
 19.05
 \end{array}$$

Fluency variation, different ways to ask children to solve 6×23 :



?

With the counters, prove that $6 \times 23 = 138$

Why is $6 \times 23 = 32 \times 6$?

Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week?

Tom saved 23p three days a week. How much did he save in 2 weeks?

Find the product of 6 and 23

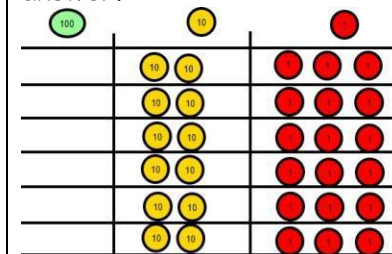
$$6 \times 23 =$$

$$= 6 \times 23$$

$$\begin{array}{r}
 6 \\
 \times 23 \\
 \hline
 \end{array}$$

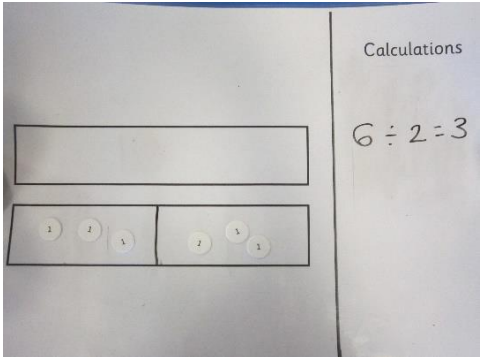
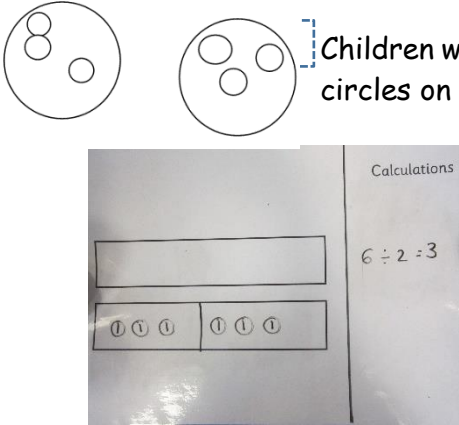

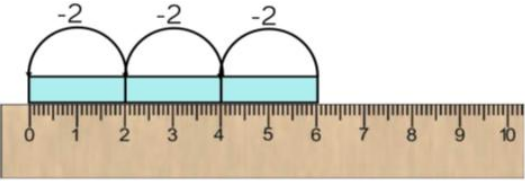
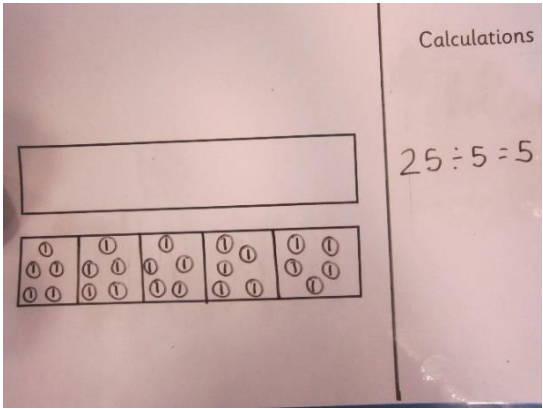
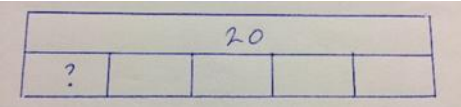
$$\begin{array}{r}
 23 \\
 \times 6 \\
 \hline
 \end{array}$$

What's the calculation? What's the answer?

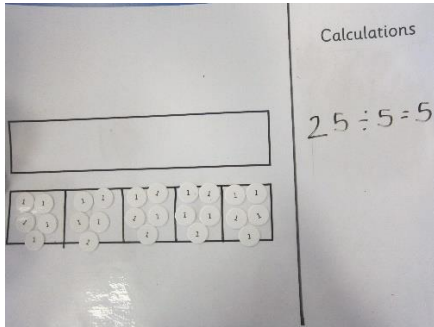


Division-

Key language which should be used: share, group, divide, divided by, half, 'is equal to' 'is the same as'

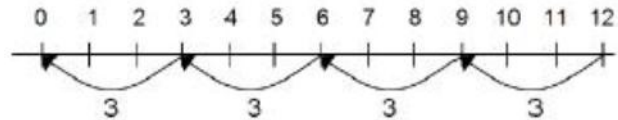
Concrete	Pictorial	Abstract										
<p>6 shared between 2 show on bar model using concrete resources</p> 	<p>Children will begin draw circles on bar model</p> 	<p>$6 \div 2 = 3$</p> <p>What's the calculation?</p> <table border="1" data-bbox="1398 583 1845 651"> <tr> <td>3</td> <td>3</td> </tr> </table>	3	3								
3	3											
<p>Understand division as repeated grouping Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding</p>   <p>3 groups of 2</p>	<p>Children will draw counters on the bar.</p> 	<p>Children move towards putting numbers on the bar model and using times table knowledge.</p> <table border="1" data-bbox="1409 1019 1875 1182"> <tr> <td colspan="5">25</td> </tr> <tr> <td>5</td> <td>5</td> <td>5</td> <td>5</td> <td>5</td> </tr> </table> <p>When children have a secure knowledge of their times tables they will be able to answer these style of questions mentally without the need to show on a bar.</p>  <p>$20 \div 5 = ?$</p> <p>$5 \times ? = 20$</p>	25					5	5	5	5	5
25												
5	5	5	5	5								

Sam wants to pack cakes into boxes of 5. How many boxes will he need for 25 cakes?



Here we are counting in 5s up to 25 to see how many lots of 5.

Use a number line to show jumps in groups. The number of jumps equals the number of groups.



$$7 \times 4 = 28$$

$$4 \times 7 = 28$$

$$28 \div 7 = 4$$

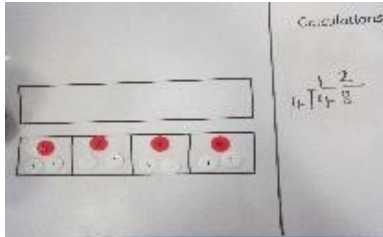
$$28 \div 4 = 7$$

Find the inverse of multiplication and division sentences by creating four linking number sentences.

2d divided by 1d using place value counters (no remainders) SHARING

done on a bar model. $48 \div 4 = 12$

Start with the tens and show calc alongside using bus stop method.

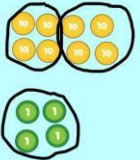


This moves on to partitioning to divide, use the grouping method, how many groups of 4 can I make. Use knowledge of x tables for multiples of ten e.g. 2 groups of 4 in 8 so 20 in 80.

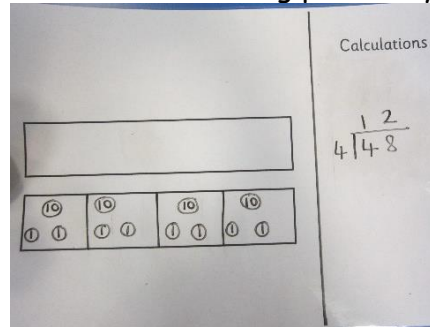
$$84 \div 4 = 21$$

$$80 \div 4 = 20$$

$$4 \div 4 = 1$$



Children to represent the place value counters and sharing pictorially on bar model.



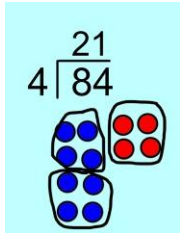
Partitioning method to be used and drawing own counters as seen in concrete image to the left.

Children will use their times table knowledge where appropriate or will show on bus stop method through abstract calculation.

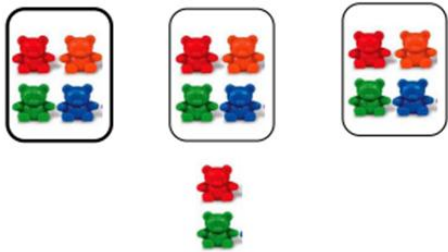
$$\begin{array}{r} 32 \\ 3 \overline{)96} \end{array}$$

Partitioning method used but children will use knowledge of their times tables to find answers before recombining.

Next stage would be using bus stop but with grouping counters under the bus stop. Use language of how many groups of 4 can I get from 8 tens? Group the tens into 4s and record number of groups above on bus stop. Repeat for ones.



2d ÷ 1d with remainders



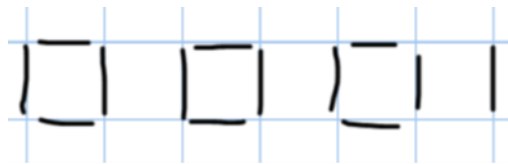
13 ÷ 4 =

Use of lollipop sticks to form wholes - squares are made a dividing by 4



Children can then draw the groups that can be made pictorially under the bus stop.

$13 \div 4 = 3 \text{ remainder } 1$



This method can then be repeated with 3 and 4 digit numbers. It can also be done with decimal places if you have a remainder.

After lots of practical experience, children will naturally move away from the need to draw groupings as they understand the method and will complete the bus stop method abstractly using times table knowledge. Showing carried over digits.

$$\begin{array}{r} 19 \\ 5 \overline{) 95} \end{array}$$

$$\begin{array}{r} 123 \\ 5 \overline{) 615} \end{array}$$

$$\begin{array}{cccc} 29 \div 8 = 3 \text{ REMAINDER } 5 \\ \uparrow \quad \uparrow \quad \uparrow \quad \quad \uparrow \\ \text{dividend} \quad \text{divisor} \quad \text{quotient} \quad \quad \text{remainder} \end{array}$$

converting remainders into decimals using bus stop method.

All whole numbers could be written with decimal and zeros e.g. 36 can be written as 36.000. If a remainder is left at the end of a calculation, add the decimal with zero and carry over the remainder to

Remainders can also be shown grouping under the bus stop. What cannot fit into a group of 4 needs carrying over and written on the bus stop so now can exchange those 3 tens for 30 ones. Becomes how many groups of 4 from 32.

Short division

Use place value counters to divide using the bus stop method alongside

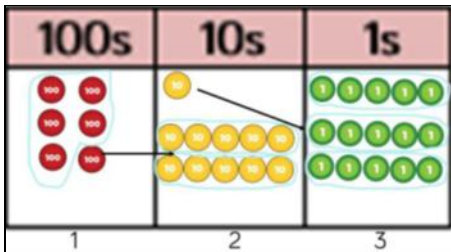
$$615 \div 5 = 123$$

create a decimal answer.

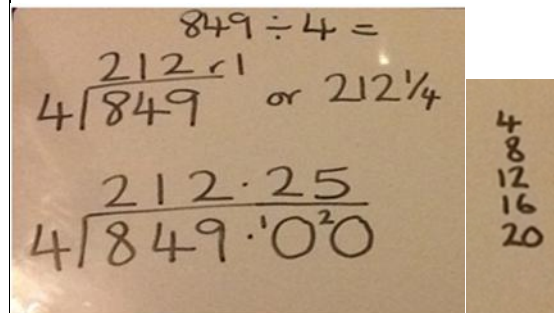
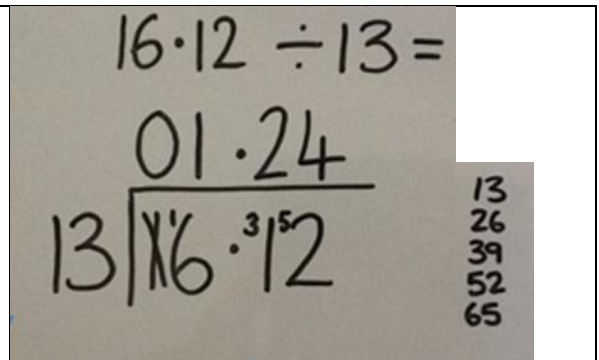
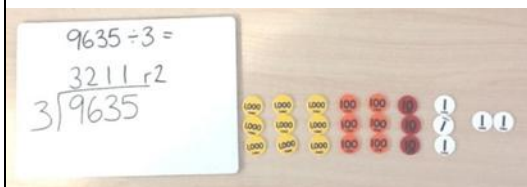
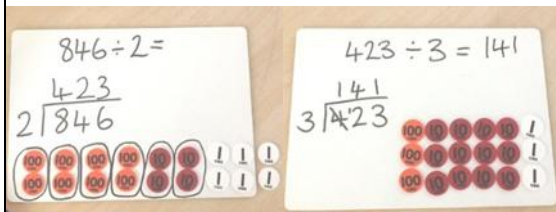
$$5 \overline{) 64} \text{ r } 4$$

$$5 \overline{) 64.0} = 12.8$$

- 5
- 10
- 15
- 20



1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?



Long division

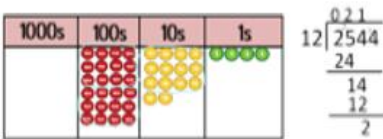
$$2544 \div 12 =$$



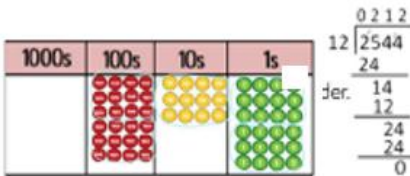
we can't group 2 thousand into 12s so we will exchange them.



We can group 24 hundreds into groups of 12 which leaves 1 hundred.



After exchanging the hundred, we have 14 tens. We can group them into 12s and have 2 tens left over.



After exchanging the 2 tens, we have 24 ones. We can group them into 2 groups of 12 which leaves no remainders.

$$\begin{array}{r} 0 \\ 12 \overline{) 2544} \end{array}$$

Step one- exchange 2 thousand for 20 hundreds so we now have 25 hundreds.

$$\begin{array}{r} 02 \\ 12 \overline{) 2544} \\ \underline{- 24} \\ 1 \end{array}$$

Step two- How many groups of 12 can I make with 25 hundreds? The 24 shows the hundreds we have grouped. The one is how many hundreds we have left.

$$\begin{array}{r} 021 \\ 12 \overline{) 2544} \\ \underline{- 24} \\ 14 \\ \underline{- 12} \\ 2 \end{array}$$

Exchange the one hundred for 10 tens. How many groups of 12 can I make with 14 tens?

The 14 shows how many tens I have, the 12 is how many I grouped and the 2 is how many tens I have left.

$$\begin{array}{r} 0212 \\ 12 \overline{) 2544} \\ \underline{- 24} \\ 14 \\ \underline{- 12} \\ 24 \\ \underline{- 24} \\ 0 \end{array}$$

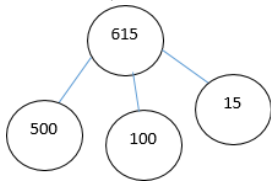
Exchange the 2 tens for 20 ones. The 24 is how many ones I have grouped and the 0 is what I have

$$\begin{array}{r} 212 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

$$\begin{array}{r} 212 \\ 12 \overline{) 2544} \\ 24 \\ 36 \\ 48 \end{array}$$

Fluency variation, different ways to ask children to solve $615 \div 5$:

Using the part whole model below, how can you divide 615 by 5 without using the 'bus stop' method?



I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

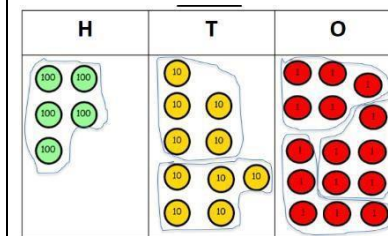
$$5 \overline{)615}$$

$$615 \div 5 =$$

$$= 615 \div 5$$

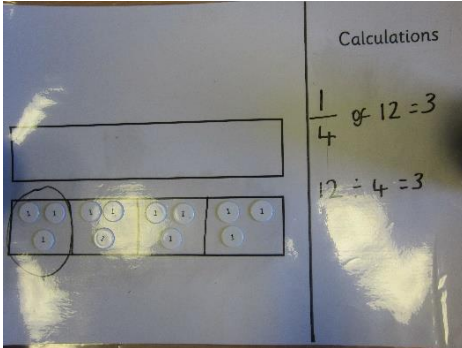
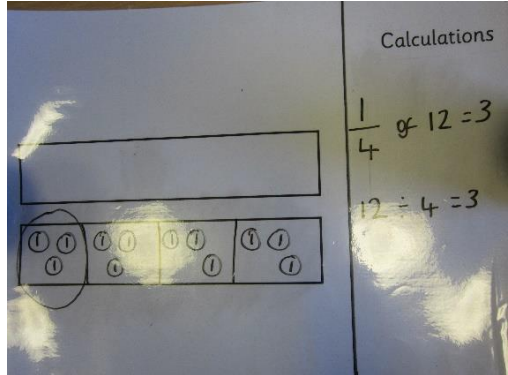
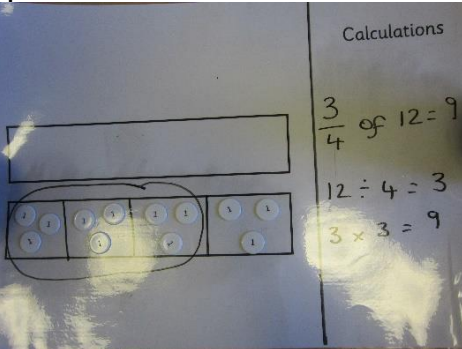
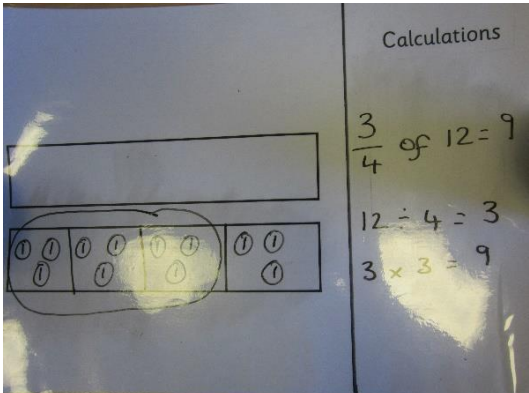
How many 5's go into 615?

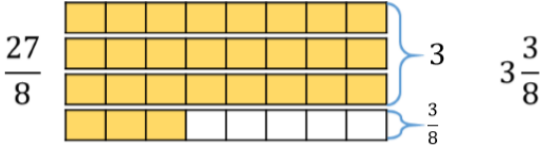
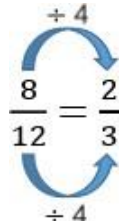
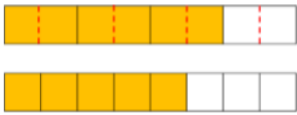
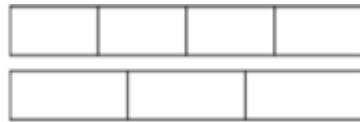

What's the calculation? What's the answer?



Fractions, Decimals and Percentages

It is a non-negotiable that bar modelling be used as an introduction to fractions and carried on being used until children are fully secure with the abstract method.

Concrete	Pictorial	Abstract
<p>Finding a fraction of an amount e.g. $\frac{1}{4}$ of 12.</p>  <p>A bar model showing a bar divided into 4 equal parts. Below the bar, 12 counters are arranged in a row, grouped into 4 groups of 3. The first group is circled. To the right, under the heading 'Calculations', the following is written: $\frac{1}{4}$ of 12 = 3 and $12 \div 4 = 3$.</p>	<p>Children will draw counters on the bar.</p>  <p>A bar model showing a bar divided into 4 equal parts. Below the bar, 12 counters are arranged in a row, grouped into 4 groups of 3. The first group is circled. To the right, under the heading 'Calculations', the following is written: $\frac{1}{4}$ of 12 = 3 and $12 \div 4 = 3$.</p>	<p>Eventually children will recognise that $\frac{1}{4}$ is dividing by 4 and use their x table knowledge. $\frac{1}{4}$ of 12 = $12 \div 4 = 3$</p>
<p>Children will move onto finding more than one part. The bar model will help to focus them on how many parts to look at.</p>  <p>A bar model showing a bar divided into 4 equal parts. Below the bar, 12 counters are arranged in a row, grouped into 4 groups of 3. The first three groups are circled. To the right, under the heading 'Calculations', the following is written: $\frac{3}{4}$ of 12 = 9, $12 \div 4 = 3$, and $3 \times 3 = 9$.</p>	<p>Children will draw counters on the bar.</p>  <p>A bar model showing a bar divided into 4 equal parts. Below the bar, 12 counters are arranged in a row, grouped into 4 groups of 3. The first three groups are circled. To the right, under the heading 'Calculations', the following is written: $\frac{3}{4}$ of 12 = 9, $12 \div 4 = 3$, and $3 \times 3 = 9$.</p>	<p>Children will divide by the fraction amount then x by how many parts. (This is quite a complex abstract method so should be used only when full understanding is evident).</p>

Learning objective	Concrete or Pictorial	Abstract
Changing fractions from improper to mixed number and vice versa		$\frac{27}{8} = 27 \div 8 = 3 \text{ r } 3 = 3\frac{3}{8}$ $3\frac{3}{8} = 3 \times 8 + 3 = \frac{27}{8}$
Use of factors to simplify fractions	A use of a multiplication grid for children to find these if they are not confident with their times table knowledge.	
Comparing and ordering fractions with multiples of the same denominator Comparing and ordering with different denominators	<p>Use bar models to compare $\frac{5}{8}$ and $\frac{3}{4}$</p>  $\frac{3}{4}$ and $\frac{2}{3}$ 	<p>Finding a common denominator (in this case 8) and using the method of equivalent fractions.</p> $\frac{3}{4} = \frac{6}{8} \quad \text{because } 4 \times 2 = 8 \text{ so repeated with the numerator, } 3 \times 2 = 6$ <p>Dora is comparing $\frac{5}{6}$ and $\frac{3}{4}$ by finding the lowest common multiple of the denominators.</p> <p>Multiples of 6: 6, 12, 18, 24</p> <p>Multiples of 4: 4, 8, 12, 16</p> <p>12 is the LCM of 4 and 6</p> $\frac{5}{6} = \frac{10}{12} \quad \frac{3}{4} = \frac{9}{12}$ $\frac{10}{12} > \frac{9}{12}$ 

Adding and subtracting fractions with the same denominator

Jess is using counters to add fractions

Complete the number sentence:

$$\underline{2} + \underline{4} =$$

$$\underline{10} \quad \underline{10}$$

$$\underline{2} + \underline{4} =$$

$$\underline{10} \quad \underline{10}$$

Adding and subtracting fractions with different denominators

Step 1	Step 2	Step 3																																									
<table border="1"> <tr> <td>$\frac{1}{3}$</td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> </tr> </table>	$\frac{1}{3}$						<table border="1"> <tr> <td>$\frac{1}{3}$</td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> </table>	$\frac{1}{3}$															<table border="1"> <tr> <td>$\frac{1}{3}$</td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> </table>	$\frac{1}{3}$																			
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Find a common denominator using the method above and change the fractions accordingly. Then add or subtract them. If the answer is an improper fraction, use the method shown above to convert it into a mixed number.

Multiplying fractions by whole numbers

$$\frac{1}{6} \times 4$$

1 sixth
1 sixth
1 sixth
1 sixth

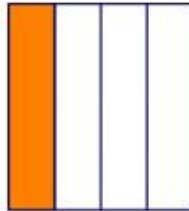
4 sixths

Denominator stays the same and you multiply the numerator by the whole number. If the answer is an improper fraction, then use the method shown above to convert it to a mixed number.

Multiplying pairs of fractions

What is $\frac{1}{3} \times \frac{1}{4}$?

This is $\frac{1}{4}$ of a rectangle.



What does $\frac{1}{3} \times \frac{1}{4}$ mean?

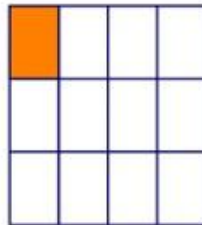
Remember $\frac{1}{3} \times \frac{1}{4}$ means:

$\frac{1}{3}$ lots of $\frac{1}{4}$

or $\frac{1}{3}$ of $\frac{1}{4}$

What is $\frac{1}{3} \times \frac{1}{4}$?

This is $\frac{1}{3}$ of our $\frac{1}{4}$ of a rectangle.



What fraction are we left with?

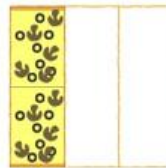
It is $\frac{1}{12}$ of the total rectangle.

$$\frac{1}{3} \times \frac{1}{4} = \frac{1 \times 1}{3 \times 4}$$

Dividing proper fractions by whole numbers

What is $\frac{1}{3} \div 2$?

This is $\frac{1}{3}$ of a pizza.



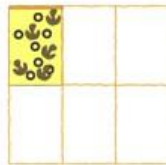
What does $\frac{1}{3} \div 2$ mean?

It means divide the $\frac{1}{3}$ into 2 equal pieces.

This is $\frac{1}{3} \div 2$

What fraction is this part?

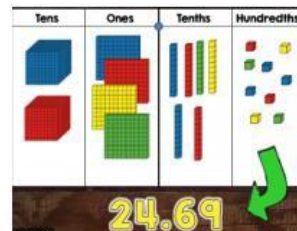
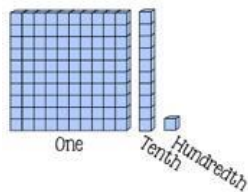
It is $\frac{1}{6}$ of the whole pizza.



$$\frac{1}{3} \div 2 = \frac{1}{2} \text{ of } \frac{1}{3} = \frac{1}{2} \times \frac{1}{3} = \frac{1 \times 1}{2 \times 3} = \frac{1}{6}$$

Recognising tenths, hundredths and thousandths

Use base ten and place value counters.



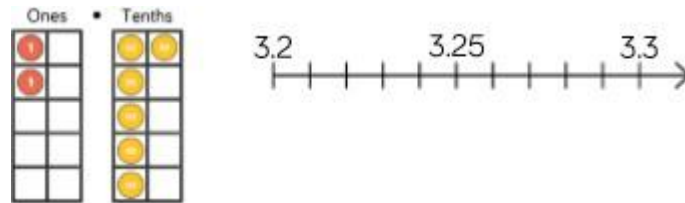
= 1 whole = 1 tenth = 1 hundredth = 1 thousandth



Representing decimal numbers

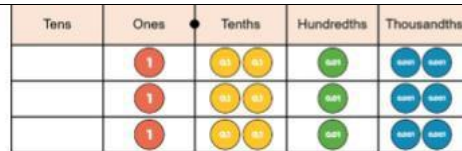
Concrete	Decimal	Decimal - expanded form	Fraction	Fraction - expanded form	In words
	3.24	$3 + 0.2 + 0.04$	$3 \frac{24}{100}$	$3 + \frac{2}{10} + \frac{4}{100}$	Three ones, two tenths and four hundredths.
	3.01		$3 \frac{1}{100}$		
				$3 + \frac{4}{10} + \frac{2}{100}$	
					Two ones, three tenths and two hundredths.

Rounding decimals

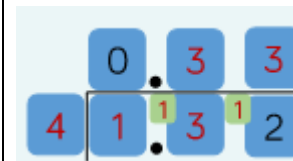
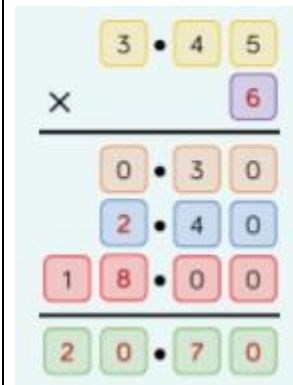
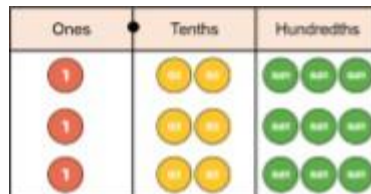


Multiplying and dividing a number with up to 3 decimal places

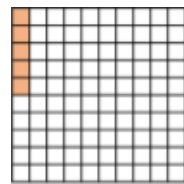
1.212 by 3



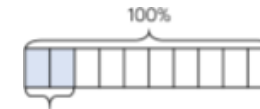
3.69 by 3



Understanding percentages as fractions



Pictorial	Parts per hundred	Percentage
	There are 51 parts per hundred.	
		75%



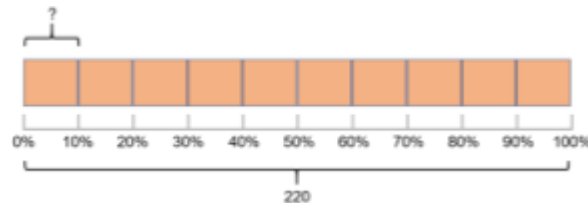
Understanding equivalent fractions, decimals and percentages

Pictorial	Percentage	Fraction	Decimal
	41 parts per hundred 41%	41 out of 100 $\frac{41}{100}$	41 hundredths 0.41
	7 parts per hundred 7%		

Decimal	Fraction	Percentage
0.35	$\frac{35}{100}$	35%
0.27		
0.6		
0.06		

Finding percentages of amounts

30% of 220



10% of 220 = 22 , so 30% of 220 = 3 × 22 = 66

For further fraction, decimal and percentages work in KS2, if children are finding the abstract difficult refer to these stages of showing fractions concrete and pictorial using a bar model.